Approximate verification and enumeration problems

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Separation of systems

Representation by polytopes

The (randomized) polytope separator

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Polynomial representation for the automaton A:

$$P_A = \sum_{w=\sigma_1 \ldots \sigma_n \in \Sigma^n} A(w) X_{1,\sigma_1} X_{2,\sigma_2} \ldots X_{n,\sigma_n}$$

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$H = \{\mathsf{ustat}_k(w) \mid A \text{ accepts } w\}$.

A accepts $a(ab)^+a^+$, that is $w = a(ab)^m b^n$.
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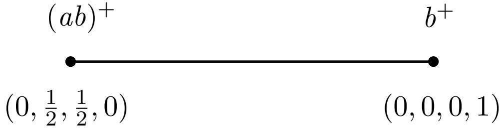

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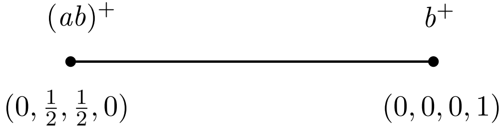

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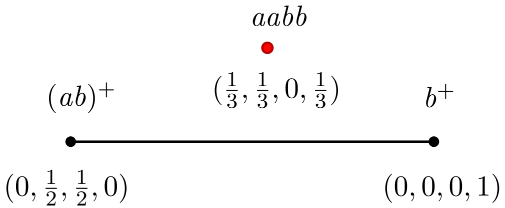

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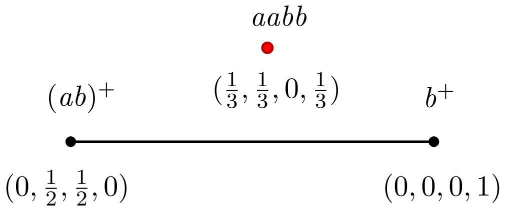

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The edit distance with moves between v and w, denoted by $d(v, w)$, is the minimal number of operations to transform v into w.

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Two polytopes K_1 and K_2 given as SMOs.

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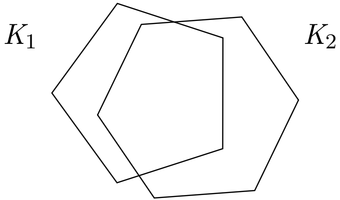

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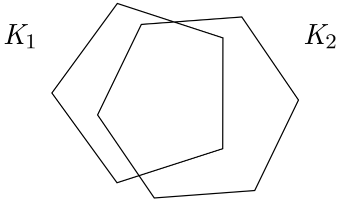

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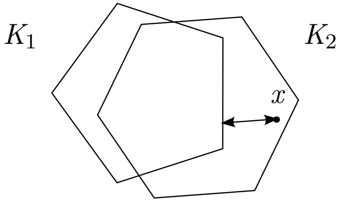

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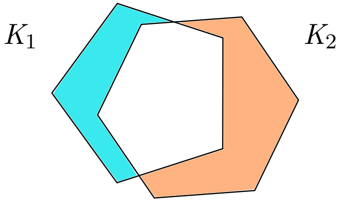

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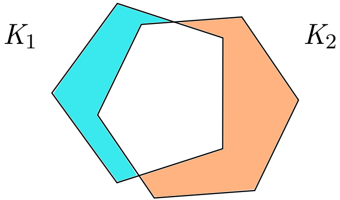

A solution for $\mathcal{H}$-polytopes

K_1, K_2 $\mathcal{H}$-polytopes: $\begin{cases} K_1 = \cap_i E_i \\ K_2 = \cap_j F_j \end{cases}$

each E_i or F_j is a half space given by a linear inequality.

Let $\bar{E}$ be the complement of the halfspace E.

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Stronger aim : compute the Hausdorff distance between two polytopes K_1, K_2.

$$d_h(K_1, K_2) = \max_{x \in K_1} d(x, K_2)$$

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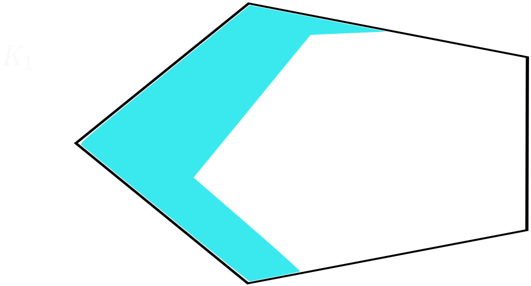

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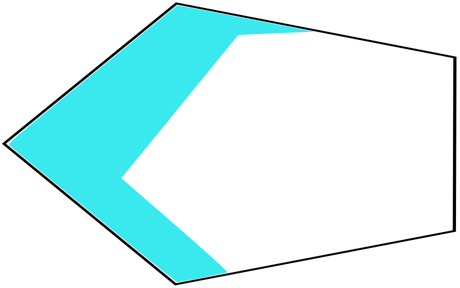

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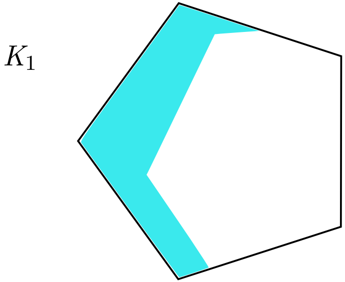

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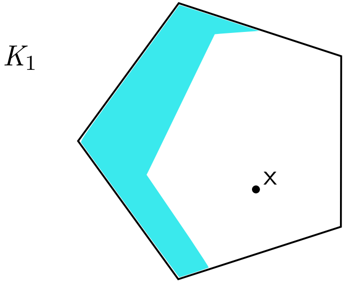

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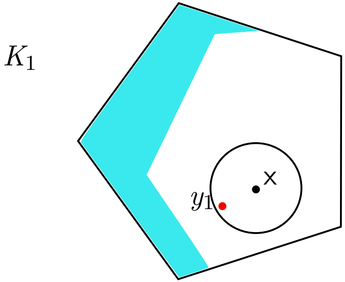

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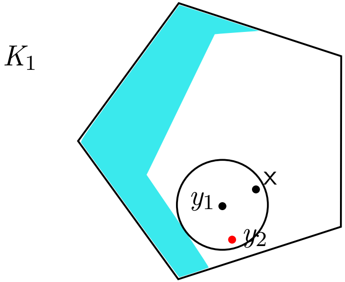

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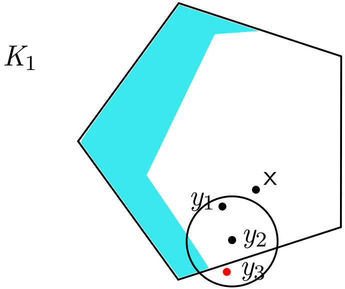

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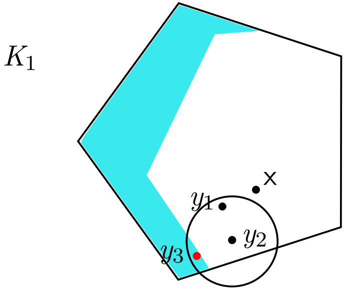

Polytope separator

Theorem

Let K_1 and K_2 be two polytopes of dimension n, given as SMOs. The Polytope Separator *algorithm outputs a point x such that $x \in K_1 \triangle K_2$ with probability greater than $2/3$. Moreover, the running time of this algorithm is polynomial in n and $d_{vol}(K_1, K_2)^{-1}$.*

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Thank you!

Questions?

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